Domain Adaptation

**Setup**

- **Source labelled samples**: \((x_S^i, y_S^i)_i \sim p_S(X, Y)\)
- **Target unlabelled samples**: \((x_T^j)_j \sim p_T(X)\)
- **Objective**: Learning \(h \in \mathcal{H}\) s.t.: \(h \in \arg\min_{h \in \mathcal{H}} \mathcal{E}_T(h)\)
Covariate Shift

Labelling functions are conserved:

\[ p_S(y|x) = p_T(y|x) \]  \hspace{1cm} (1)
Covariate Shift

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\[ p_S(y|x) = p_T(y|x) \]  \hspace{1cm} (1)

\[ \varepsilon_T(h) = \mathbb{E}_T[\ell(h(x), y)] = \mathbb{E}_S \left[ \frac{p_T(x)}{p_S(x)} \ell(h(x), y) \right] \]  \hspace{1cm} (2)
Covariate Shift

Labelling functions are conserved:

$$p_S(y|x) = p_T(y|x)$$  \hspace{1cm} (1)

$$\varepsilon_T(h) = \mathbb{E}_T[\ell(h(x), y)] = \mathbb{E}_S \left[ \frac{p_T(x)}{p_S(x)} \ell(h(x), y) \right]$$  \hspace{1cm} (2)

Needs overlapping supports!
Domain Invariant Representations [Ben-David et al., Ganin et al.]

$p_T(x)$

Non-overlapping distributions

$p_S(x)$
Domain Invariant Representations [Ben-David et al., Ganin et al.]

$p_T(x)$
Non-overlapping distributions

$p_S(x)$

Deep net
Domain Invariant Representations [Ben-David et al., Ganin et al.]

- $p_T(x)$
- $p_T(z)$
- $p_S(x)$
- $p_S(z)$

Deep net

Non-overlapping distributions

Overlapping distributions
Domain Invariant Representations [Ben-David et al., Ganin et al.]

Non-overlapping distributions

Overlapping distributions

Deep net

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[ p_T(x) \]

Non-overlapping distributions

\[ p_S(x) \]

Deep net

Overlapping distributions

\[ p_T(z) \]

\[ p_S(z) = p_T(z) \]

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

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Deep net

\[ p_T(z) \]

\[ p_S(z) \]

Representation \( \varphi \in \Phi \)

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[ p_T(x) \quad p_T(z) \quad p_S(x) \quad p_S(z) \]

Non-overlapping distributions

Deep net

Representation

\[ \varphi \in \Phi \]

Classifier

\[ g \in G \]

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

Non-overlapping distributions

Representation

Classifier

Bike?

Err Target ≤
Domain Invariant Representations [Ben-David et al., Ganin et al.]

Non-overlapping distributions

\[ \varphi \in \Phi \]

Deep net

\[ g \in \mathcal{G} \]

\[ p_T(x) \]

\[ p_T(z) \]

\[ p_S(x) \]

\[ p_S(z) \]

Bike?

Err Target \( \leq \) Err Source
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[ p_T(x) \quad \text{pT(z)} \quad p_S(x) \quad p_S(z) \]

Non-overlapping distributions

Deep net

Representation \( \varphi \in \Phi \)

Classifier \( g \in G \)

Err Target \( \leq \) Err Source + Distribution divergence

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[ \varphi \in \Phi \]

\[ g \in \mathcal{G} \]

\[ p_T(x) \]

\[ p_T(z) \]

\[ p_S(x) \]

\[ p_S(z) \]

Non-overlapping distributions

Deep net

Representation

Classifier

Err Target \leq Err Source + Distribution divergence + ?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[
\begin{align*}
\varphi & \in \Phi \\
p_T(x) & \quad \text{Non-overlapping distributions} \\
p_S(x) & \\
\text{Err Target} & \leq \text{Err Source} + \text{Distribution divergence} + ?
\end{align*}
\]
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[
p_T(x) \quad \varphi \in \Phi \quad p_T(z) \quad g \in \mathcal{G} \quad p_S(x) \quad p_S(z)
\]

Non-overlapping distributions

Representation

Classifier

\[\text{Err Target} \leq \text{Err Source} + \text{Distribution divergence} + \?\]

Controllable at train time

Bike?
Domain Invariant Representations [Ben-David et al., Ganin et al.]

\[ p_T(x) \quad p_T(z) \quad p_S(x) \quad p_S(z) \]

Non-overlapping distributions

Representation

\[ \varphi \in \Phi \]

Classifier

\[ g \in \mathcal{G} \]

Deep net

\[ p_T(z) \quad p_S(z) \]

Bike?

\[ \text{Err Target} \leq \text{Err Source} + \text{Distribution divergence} + \text{Adaptability} \]

Controllable at train time

\[ \inf_{g \in \mathcal{G}} \varepsilon_S(g\varphi) + \varepsilon_T(g\varphi) \]
Domain Invariant Representations: Limits

\[
\varepsilon_T(g\varphi) \leq \varepsilon_S(g\varphi) + d_G(\varphi) + \lambda_G(\varphi) \tag{3}
\]

**An unexpected trade-off**

Let \(\psi\) be a representation which is a richer feature extractor than \(\varphi\):

\[
G \circ \varphi \subseteq G \circ \psi
\]
Domain Invariant Representations: Limits

\[ \varepsilon_T(g\varphi) \leq \varepsilon_S(g\varphi) + d_G(\phi) + \lambda_G(\phi) \]  \hspace{1cm} (3)

An unexpected trade-off

Let \( \psi \) be a representation which is a richer feature extractor than \( \varphi \):

\[ G \circ \varphi \subset G \circ \psi \]

Then,

\[ d_G(\varphi) \leq d_G(\psi) \ \text{while} \ \lambda_G(\psi) \leq \lambda_G(\varphi) \]  \hspace{1cm} (4)
\[ \epsilon_T(g\varphi) \leq \underbrace{\epsilon_S(g\varphi) + d_G(\varphi)}_{\text{Controllable}} + \underbrace{\lambda_G(\varphi)}_{\text{Not controllable}} \]  

**An unexpected trade-off**

Let \( \psi \) be a representation which is a richer feature extractor than \( \varphi \):

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▷ The benefit of representation invariance must be higher than the loss of adaptability, which is impossible to guarantee in practice.
Domain Invariant Representations: Limits

\[ \varepsilon_T(g\varphi) \leq \varepsilon_S(g\varphi) + d_G(\varphi) + \underbrace{\lambda_G(\varphi)}_{\text{Not controllable}} \]  

(3)

**An unexpected trade-off**

Let \( \psi \) be a representation which is a richer feature extractor than \( \varphi \):

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Then,

\[ d_G(\varphi) \leq d_G(\psi) \quad \text{while} \quad \lambda_G(\psi) \leq \lambda_G(\varphi) \]  

(4)

▷ The benefit of representation invariance must be higher than the loss of adaptability, which is impossible to guarantee in practice.

**Invariance is conflicting with label shift [Zhao et al., ICML2019]**

\[ \lambda_G(\varphi) \geq \frac{1}{2} (JS(Y) - JS(Z))^2 \]  

(5)

If \( JS(Z) \to 0 \), \( \lambda_G(\varphi) \) can not be small if \( JS(Y) \) is high...
Outline
1. A new trade-off between invariance and transferability
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   - Introduce a new error term named transferability error
   - Reconcile Invariant Representations and Weights
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   - Reconcile Invariant Representations and Weights

2. Role of **Inductive Bias**:
   - Weights \(\triangleright\) *induce new property of invariance of representations and the labelling function*
   - Classifier \(\triangleright\) Feedback for better representations invariance.
1. A new trade-off between invariance and transferability
   - Introduce a new error term named *transferability error*
   - Reconcile Invariant Representations and Weights

2. Role of **Inductive Bias**:  
   - Weights ▷ *induce new property of invariance of representations and the labelling function*
   - Classifier ▷ Feedback for better representations invariance.

3. A new algorithm for Robust Unsupervised Domain Adaptation ▷ RUDA
   - Robust to strong label shift
   - Evaluation on two benchmarks
A new trade-off between invariance and transferability
Three ingredients

1. **INV** captures the difference between source and target distribution of representations.
Three ingredients

1. **INV** ▷ captures the difference between source and target distribution of representations.

2. **TSF** ▷ catches if the coupling between representations and labels shifts across domains.
Three ingredients

1. **INV** Λ captures the difference between source and target distribution of representations.
2. **TSF** Λ catches if the coupling between representations and labels shifts across domains.
3. Reconcile Weights and Invariant Representations.
A new trade-off between invariance and transferability
A new trade-off between invariance and transferability

\[
INV(\varphi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[f(Z)] - \mathbb{E}_T[f(Z)]
\]

captures the difference between source and target distribution of representations
A new trade-off between invariance and transferability

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captures the difference between source and target distribution of representations.
A new trade-off between invariance and transferability

$$TSF(\varphi) := \sup_{f \in \mathcal{F}_c} E_S[Y \cdot f(Z)] - E_T[Y \cdot f(Z)]$$

Catches if the coupling between representations and labels shifts across domains.
A new trade-off between invariance and transferability

\[
\text{TSF}(\phi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)]
\]

catches if the coupling between representations and labels shifts across domains

Non-overlapping distributions

Deep net

Bike? Backpack?
A new trade-off between invariance and transferability

$$\text{TSF}(\varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)]$$

catches if the coupling between representations and labels shifts across domains
A new trade-off between invariance and transferability

\[ TSF(\varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] \]

catches if the coupling between representations and labels shifts across domains

\( f(\text{Bike?}) = (0, 1) \)

Non-overlapping distributions

Deep net
A new trade-off between invariance and transferability

\[ \text{TSF}(\phi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] \]

Catches if the coupling between representations and labels shifts across domains.

\( f(\text{Bike}) = (0, 1) \cdot \text{Bike?} = 0 \)

Non-overlapping distributions

Deep net

Bike?

Backpack?
A new trade-off between invariance and transferability

\[ \operatorname{TSF}(\varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] \]

Catches if the coupling between representations and labels shifts across domains.

\[ f(\text{Bike?}) = (0, 1) \]
\[ f(\text{Backpack?}) = 0 \]
\[ f(\text{Bike?}) = 1 \]

Non-overlapping distributions
Reconciling Weights and Invariant Representations
Reconciling Weights and Invariant Representations

$p_T(x)$

$p_S(x)$

Non-overlapping distributions

Deep net
Reconciling Weights and Invariant Representations

$p_T(x)$  
Non-overlapping distributions

$p_T(z)$

$p_S(x)$

$p_S(z)$

Deep net
Reconciling Weights and Invariant Representations

$p_T(x)$

Non-overlapping distributions

$p_S(x)$

Deep net

$p_T(z)$

xBike?

$p_S(z)$

Backpack?

$w(z)$

Deep net
Reconciling Weights and Invariant Representations

$p_T(x)$

Non-overlapping distributions

$p_S(x)$

$p_T(z)$

$p_S(z)$

Deep net

$w(z)$

$\times 1$

$\times 2$

$\times 0$

Bike?

Backpack?
Reconciling Weights and Invariant Representations

\[
p_T(x) \quad \text{Non-overlapping distributions} \quad p_S(x)
\]

Deep net

\[
p_T(z) \quad w(z) \quad p_S(z)
\]

Bike? Backpack?

\[
\times 1 \times 2 \times 0
\]

\[
\text{INV}(w, \varphi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[w(Z)f(Z)] - \mathbb{E}_T[f(Z)]
\]
Reconciling Weights and Invariant Representations

\[
\begin{align*}
\mathbb{P}_T(x) & \quad \mathbb{P}_T(z) \\
\mathbb{P}_S(x) & \quad \mathbb{P}_S(z)
\end{align*}
\]

Non-overlapping distributions

Deep net

\[
\begin{align*}
\text{Bike?} & \quad \times 1 \\
\text{Backpack?} & \quad \times 2 \\
\end{align*}
\]

\[
\begin{align*}
\text{INV}(w, \varphi) & := \sup_{f \in \mathcal{F}} \mathbb{E}_S[w(Z)f(Z)] - \mathbb{E}_T[f(Z)] \\
\text{TSF}(w, \varphi) & := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[w(Z)Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)]
\end{align*}
\]
Reconciling Weights and Invariant Representations

$p_T(x)$

$p_T(z)$

$p_S(x)$

$p_S(z)$

Non-overlapping distributions

Deep net

$INV(w, \varphi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[w(Z)f(Z)] - \mathbb{E}_T[f(Z)]$

$TSF(w, \varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[w(Z)Y \cdot f(Z)] - \mathbb{T}[Y \cdot f(Z)]$

$\varepsilon_T(g \varphi) \leq \varepsilon_{w,S}(g \varphi)$

Bike?

Backpack?

$\times 1$

$\times 2$

$\times 0$
Reconciling Weights and Invariant Representations

$\text{INV}(w, \varphi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[w(Z)f(Z)] - \mathbb{E}_T[f(Z)]$

$\text{TSF}(w, \varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[w(Z)Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)]$

$\epsilon_T(g\varphi) \leq \epsilon_{w,S}(g\varphi) + 6 \cdot \text{INV}(w, \varphi)$
Reconciling Weights and Invariant Representations

Let $p_T(x)$ represent the distribution of images of bikes, and $p_S(x)$ represent the distribution of images of backpacks. The deep net architecture is trained to distinguish between these two classes. However, due to the non-overlapping nature of these distributions, the model may struggle to learn invariant representations.

To address this, we define a weight function $w(z)$ that captures the relevance of each feature $z$ to the task of distinguishing between bikes and backpacks. This function can be used to reweight the model's predictions, allowing it to better focus on the most informative features.

The weight function is defined as:

$$ w(z) = \sum_{i=1}^{n} \alpha_i z_i $$

where $\alpha_i$ are the learned weights and $z_i$ are the feature values.

The final output of the model is then given by:

$$ f(x) \times w(z) $$

This approach allows the model to effectively balance the weights between the two classes, leading to improved performance.

Formally, we define:

$$ INV(w, \varphi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[w(Z)f(Z)] - \mathbb{E}_T[f(Z)] $$

and

$$ TSF(w, \varphi) := \sup_{f \in \mathcal{F}_e} \mathbb{E}_S[w(Z)Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] $$

where $\mathcal{F}$ and $\mathcal{F}_e$ are subsets of the feature space.

Using these quantities, we have:

$$ \varepsilon_T(g\varphi) \leq \varepsilon_{w,S}(g\varphi) + 6 \cdot INV(w, \varphi) + 2 \cdot TSF(w, \varphi) $$

This inequality provides a bound on the generalization error, giving us a way to evaluate the effectiveness of our approach.
Reconciling Weights and Invariant Representations

\[ p_T(x) \]

Non-overlapping distributions

\[ p_S(x) \]

Deep net

\[ p_T(z) \]

\[ p_S(z) \]

\[ w(z) \]

\[ \text{Bike?} \]

\[ \text{Backpack?} \]

\[ \times 1 \]

\[ \times 2 \]

\[ \times 0 \]

\[ \text{INV}(w, \varphi) := \sup_{f \in \mathcal{F}} E_S[w(Z)f(Z)] - E_T[f(Z)] \]

\[ \text{TSF}(w, \varphi) := \sup_{f \in \mathcal{F}_\varepsilon} E_S[w(Z)Y \cdot f(Z)] - E_T[Y \cdot f(Z)] \]

\[ \varepsilon_T(g \varphi) \leq \varepsilon_{w,S}(g \varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T \varphi) \]
\[ \varepsilon_T(g\varphi) \leq \varepsilon_w \cdot S(g\varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T\varphi) \] (6)
Reconciling Weights and Invariant Representations

\[ \varepsilon_T(g\varphi) \leq \varepsilon_{w \cdot S}(g\varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T\varphi) \] (6)

**Weighting the source domain**

\[ \varepsilon_{w \cdot S}(g\varphi) := \mathbb{E}_S[w(Z)\ell(g(Z), Y)] \] (7)

**A new trade-off**

\[ \varepsilon_T(f_T\varphi) := \inf_{f \in \mathcal{F}_c} \varepsilon_T(f\varphi) \] (8)
Reconciling Weights and Invariant Representations

\[ \varepsilon_T(g \varphi) \leq \varepsilon_{w \cdot S}(g \varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T \varphi) \quad (6) \]

**Weighting the source domain**

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**A new trade-off**

\[ \varepsilon_T(f_T \varphi) := \inf_{f \in F_c} \varepsilon_T(f \varphi) \quad (8) \]

**Tightness**

If \( \text{INV}(w, \varphi) = 0 \) and \( \text{TSF}(w, \varphi) = 0 \), then,

\[ \Rightarrow p_T(y|z) = p_S(y|z) \quad \text{and} \quad w(z) = \frac{p_T(z)}{p_S(z)} \]

▷ Much smaller than the adaptability.
Remaining challenges

\[ \varepsilon_T(g\varphi) \leq \varepsilon_w \cdot s(g\varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T \varphi) \quad (9) \]
\[ \varepsilon_T(g\varphi) \leq \varepsilon_w \cdot S(g\varphi) + 6 \cdot \text{INV}(w, \varphi) + 2 \cdot \text{TSF}(w, \varphi) + \varepsilon_T(f_T \varphi) \quad (9) \]

1. Classifier \( \triangleright \) address the lack of labelled data in the target domain.

\[ \text{TSF}(w, \varphi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[w(Z)Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] \]

2. Weights \( \triangleright \) induce invariance property on representations (see the paper for more details).
Remaining challenges

\[ \varepsilon_T(g\phi) \leq \varepsilon_w \cdot s(g\phi) + 6 \cdot \text{INV}(w, \phi) + 2 \cdot \text{TSF}(w, \phi) + \varepsilon_T(f_T \phi) \quad (9) \]

1. **Classifier** ▷ address the lack of labelled data in the target domain.

\[
\text{TSF}(w, \phi) := \sup_{f \in F_c} \mathbb{E}_S\left[w(Z)Y \cdot f(Z)\right] - \mathbb{E}_T\left[Y \cdot f(Z)\right]
\]

2. **Weights** ▷ induce invariance property on representations (see the paper for more details).
Inductive design of the classifier
Inductive design of the classifier

\[
\hat{TSF}(\varphi, \tilde{g}) := \sup_{f \in F_C} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[\tilde{g}(Z) \cdot f(Z)]
\] (10)

Insight

The best source classifier is not the best target classifier, and it is possible to improve the best source classifier, e.g., specific architecture or a well-suited regularization.

Inductive design

We say that there is an inductive design of a classifier at level $0 < \beta \leq 1$ if for any representations $\varphi$, noting $g_S = \arg \min_{g \in G} \varepsilon_S(g \varphi)$, we can determine $\tilde{g}$ such that:

\[
\varepsilon_T(\tilde{g} \varphi) \leq \beta \varepsilon_T(g_S \varphi)
\] (11)

- $\beta$ is strong when $\beta < 1$,
- $\beta$ is weak when $\beta = 1$.
Inductive design of the classifier

\[
\widehat{\text{TSF}}(\varphi, \tilde{g}) := \sup_{f \in F_C} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[\tilde{g}(Z) \cdot f(Z)]
\]  

(10)

**Insight**

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**Insight**
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**Inductive design**
We say that there is an inductive design of a classifier at level \(0 < \beta \leq 1\) if for any representations \(\varphi\), noting \(g_S = \arg \min_{g \in G} \varepsilon_S(g\varphi)\), we can determine \(\tilde{g}\) such that:

\[
\varepsilon_T(\tilde{g}\varphi) \leq \beta \varepsilon_T(g_S\varphi)
\]

- \(\beta\)—strong when \(\beta < 1\),
- weak when \(\beta = 1\).
A revisited version of the bound:

$$\varepsilon_T(g_S\varphi) \leq \varepsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(\tilde{g}\varphi) + \varepsilon_T(f_T\varphi) \quad (12)$$
Generalization guarantees with inductive bias

A revisited version of the bound:

$$
\varepsilon_T(g_S\varphi) \leq \varepsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \widehat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(\tilde{g}\varphi) + \varepsilon_T(f_T\varphi) \quad (12)
$$

- \( \widehat{\text{TSF}}(\varphi, \tilde{g}) \) \( \triangleright \) \text{transferability of representations with respect to the inductive design:}

\[
\widehat{\text{TSF}}(\varphi, \tilde{g}) := \sup_{f \in F_C} E_S[Y \cdot f(Z)] - E_T[\tilde{g}(Z) \cdot f(Z)] \quad (13)
\]
A revisited version of the bound:

\[ \varepsilon_T(g_S \varphi) \leq \varepsilon_S(g_S \varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \text{TSF}(\varphi, \tilde{g}) + \varepsilon_T(\tilde{g} \varphi) + \varepsilon_T(f_T \varphi) \]  (12)

- \( \text{TSF}(\varphi, \tilde{g}) \) \( \triangleright \) transferability of representations with respect to the inductive design:

\[ \text{TSF}(\varphi, \tilde{g}) := \sup_{f \in F_C} \mathbb{E}_S [Y \cdot f(Z)] - \mathbb{E}_T [\tilde{g}(Z) \cdot f(Z)] \]  (13)

- \( \varepsilon_T(\tilde{g} \varphi) \) \( \triangleright \) How good is the inductive design.
Generalization guarantees with inductive bias

A revisited version of the bound:

$$\epsilon_T(g_S\phi) \leq \epsilon_S(g_S\phi) + 6 \cdot \text{INV}(\phi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \epsilon_T(\tilde{g} \phi) + \epsilon_T(f_T \phi) \tag{12}$$

- $\hat{\text{TSF}}(\varphi, \tilde{g}) \triangleright \text{transferability of representations with respect to the inductive design:}$

$$\hat{\text{TSF}}(\varphi, \tilde{g}) := \sup_{f \in F \in F_C} \mathbb{E}_{S}[Y \cdot f(Z)] - \mathbb{E}_{T}[\tilde{g}(Z) \cdot f(Z)] \tag{13}$$

- $\epsilon_T(\tilde{g} \phi) \triangleright \text{How good is the inductive design.}$

Here comes the guarantees

$$\boxed{\epsilon_T(g_S\phi) \leq \epsilon_S(g_S\phi) + 6 \cdot \text{INV}(\phi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \beta \epsilon_T(g_S\phi) + \epsilon_T(f_T \phi)}$$
Generalization guarantees with inductive bias

A revisited version of the bound:

$$\varepsilon_T(g_S \varphi) \leq \varepsilon_S(g_S \varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(\tilde{g} \varphi) + \varepsilon_T(f_T \varphi) \tag{12}$$

- $\hat{\text{TSF}}(\varphi, \tilde{g}) \triangleright$ transferability of representations with respect to the inductive design:

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- $\varepsilon_T(\tilde{g} \varphi) \triangleright$ How good is the inductive design.

Here comes the guarantees

$$\left[ \varepsilon_T(g_S \varphi) \right] \leq \frac{1}{1 - \beta} \left( \varepsilon_S(g_S \varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(f_T \varphi) \right)$$
Generalization guarantees with inductive bias

A revisited version of the bound:

$$\varepsilon_T(g_S \varphi) \leq \varepsilon_S(g_S \varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \overset{\text{TSF}}{\text{Sup}}(\varphi, \tilde{g}) + \varepsilon_T(\tilde{g} \varphi) + \varepsilon_T(f_T \varphi) \quad (12)$$

- $\overset{\text{TSF}}{\text{Sup}}(\varphi, \tilde{g}) \triangleright$ transferability of representations with respect to the inductive design:

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$$\varepsilon_T(\tilde{g} \varphi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_S(g_S \varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \overset{\text{TSF}}{\text{Sup}}(\varphi, \tilde{g}) + \varepsilon_T(f_T \varphi) \right)$$
Generalization guarantees with inductive bias

\[ \varepsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \text{TSF}(\varphi, \tilde{g}) + \varepsilon_T(f_T\varphi) \right) \]
Generalization guarantees with inductive bias

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\epsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{1 - \beta} \left( \epsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \widehat{\text{TSF}}(\varphi, \tilde{g}) + \epsilon_T(f_T\varphi) \right)
$$
\[ \varepsilon_T(\tilde{g}\phi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_s(g_s\phi) + 6 \cdot \text{INV}(\phi) + 2 \cdot \text{TSF}(\phi, \tilde{g}) + \varepsilon_T(f_T\phi) \right) \]

- Target labels are only involved in \( \varepsilon_T(f_T\phi) \) which reflects the level of noise when fitting labels from representations \( \triangleright \text{transferability is now free of target labels.} \)
Generalization guarantees with inductive bias

\[ \varepsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \widehat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(f_T\varphi) \right) \]

- Target labels are only involved in \( \varepsilon_T(f_T\varphi) \) which reflects the level of noise when fitting labels from representations. \(\triangleright\) transferability is now free of target labels.

- the weaker the inductive bias (\( \beta \to 1 \)), the higher the bound and vice versa.
Generalization guarantees with inductive bias

\[ \varepsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_S(g_S\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \text{TSF}(\varphi, \tilde{g}) + \varepsilon_T(f_T\varphi) \right) \]

- Target labels are only involved in \( \varepsilon_T(f_T\varphi) \) which reflects the level of noise when fitting labels from representations \( \triangleright \textit{transferability is now free of target labels.} \)
- the weaker the inductive bias \( (\beta \to 1) \), the higher the bound and vice versa.
- \textbf{Takeaways} \( \triangleright \) if a regularization is available, it will interacts with the transferability error.
Robust Unsupervised Domain Adaptation (RUDA)
Assumptions

- Weak inductive design of the classifier:
  - **Classifier**: $\tilde{g} \leftarrow g_S (\beta = 1)$
  - *No theoretical guarantees from inductive classifier.*
Assumptions

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  - **Classifier:** $\tilde{g} \leftarrow gs (\beta = 1)$
  - *No theoretical guarantees from inductive classifier.*
- Weights controls the invariance error: $w(z) = \frac{p_T(z)}{p_S(z)}$
Assumptions

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  - **Classifier**: \( \tilde{g} \leftarrow g_\beta \) \((\beta = 1)\)
  - *No theoretical guarantees from inductive classifier.*
- Weights controls the invariance error: \( w(z) = \frac{p_T(z)}{p_S(z)} \)
- Bring strong robustness to the adaptation procedure ▶ *stress-test on dataset with label strong label shift.*
Assumptions

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- Bring strong robustness to the adaptation procedure ▷ *stress-test on dataset with label strong label shift.*

\[
\varphi^* = \arg \min_{\varphi \in \Phi} \varepsilon_w(\varphi) \cdot S(g_w \cdot S \varphi) + \lambda \cdot \overline{\text{TSF}}(w, \varphi, g_w \cdot S)
\]

such that $w(\varphi) = \arg \min_w \text{INV}(w, \varphi)$

▷ More details about the procedure in the paper.
Experiments
Experiments

RUDA performs similarly than SOTA approaches

Table 1: Accuracy (%) on the Office-31 dataset.

<table>
<thead>
<tr>
<th>Method</th>
<th>A→W</th>
<th>W→A</th>
<th>A→D</th>
<th>D→A</th>
<th>D→W</th>
<th>W→D</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>ResNet-50</td>
<td>68.4 ± 0.2</td>
<td>60.7 ± 0.3</td>
<td>68.9 ± 0.2</td>
<td>62.5 ± 0.3</td>
<td>96.7 ± 0.1</td>
<td>99.3 ± 0.1</td>
<td>76.1</td>
</tr>
<tr>
<td>DANN</td>
<td>82.0 ± 0.4</td>
<td>67.4 ± 0.5</td>
<td>79.7 ± 0.4</td>
<td>68.2 ± 0.4</td>
<td>96.9 ± 0.2</td>
<td>99.1 ± 0.1</td>
<td>82.2</td>
</tr>
<tr>
<td>CDAN</td>
<td>93.1 ± 0.2</td>
<td>68.0 ± 0.4</td>
<td>89.8 ± 0.3</td>
<td>70.1 ± 0.4</td>
<td>98.2 ± 0.2</td>
<td>100. ± 0.0</td>
<td>86.6</td>
</tr>
<tr>
<td>CDAN+E</td>
<td>94.1 ± 0.1</td>
<td>69.3 ± 0.4</td>
<td>92.9 ± 0.2</td>
<td>71.0 ± 0.3</td>
<td>98.6 ± 0.1</td>
<td>100. ± 0.0</td>
<td>87.7</td>
</tr>
<tr>
<td>RUDA</td>
<td>94.3 ± 0.3</td>
<td>70.7 ± 0.3</td>
<td>92.1 ± 0.3</td>
<td>70.7 ± 0.1</td>
<td>98.5 ± 0.1</td>
<td>100. ± 0.0</td>
<td>87.6</td>
</tr>
<tr>
<td>RUDA_w</td>
<td>92.0 ± 0.3</td>
<td>67.9 ± 0.3</td>
<td>91.1 ± 0.3</td>
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</table>

<table>
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<tr>
<th>Shift of [16 ± 31]</th>
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<tr>
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</tr>
<tr>
<td>DANN</td>
</tr>
<tr>
<td>CDAN</td>
</tr>
<tr>
<td>RUDA</td>
</tr>
<tr>
<td>IWAN</td>
</tr>
<tr>
<td>CDAN_w</td>
</tr>
<tr>
<td>RUDA_w</td>
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Table 2: Accuracy (%) on the Digits dataset.

<table>
<thead>
<tr>
<th>Method</th>
<th>Shift of [0 ~ 5]</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>100%</th>
<th>Avg</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>100%</th>
<th>Avg</th>
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<tbody>
<tr>
<td>DANN</td>
<td>41.7</td>
<td>51.0</td>
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<td>94.5</td>
<td>63.2</td>
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<td></td>
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<td></td>
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<tr>
<td>CDAN</td>
<td>50.7</td>
<td>62.2</td>
<td>82.9</td>
<td>82.8</td>
<td>96.9</td>
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<td></td>
</tr>
<tr>
<td>RUDA</td>
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<td>58.4</td>
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<td>84.0</td>
<td>95.5</td>
<td>72.5</td>
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<td></td>
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<tr>
<td>IWAN</td>
<td>73.7</td>
<td>74.4</td>
<td>78.4</td>
<td>77.5</td>
<td>95.7</td>
<td>79.9</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CDAN_w</td>
<td>68.3</td>
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<td>83.4</td>
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<td>RUDA_w</td>
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</tbody>
</table>

15
Experiments

RUDA still performs well even when stressed with strong label shift.

Table 1: Accuracy (%) on the Office-31 dataset.

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<th>W→A</th>
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<tr>
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<td>99.3±0.1</td>
<td>76.1</td>
</tr>
<tr>
<td>DANN</td>
<td>82.0±0.4</td>
<td>67.4±0.5</td>
<td>79.7±0.4</td>
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<td>96.9±0.2</td>
<td>99.1±0.1</td>
<td>82.2</td>
</tr>
<tr>
<td>CDAN</td>
<td>93.1±0.2</td>
<td>68.0±0.4</td>
<td>89.8±0.3</td>
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<td><strong>87.7</strong></td>
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<tr>
<td>RUDA</td>
<td><strong>94.3±0.3</strong></td>
<td><strong>70.7±0.3</strong></td>
<td>92.1±0.3</td>
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<td>100.0±0.0</td>
<td>87.6</td>
</tr>
<tr>
<td>RUDA_w</td>
<td>92.0±0.3</td>
<td>67.9±0.3</td>
<td>91.1±0.3</td>
<td>70.2±0.2</td>
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<td>IWN</td>
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<td>97.0±0.0</td>
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<td>75.0</td>
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<td>81.5±0.5</td>
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<td><strong>100.0±0.0</strong></td>
<td><strong>83.8</strong></td>
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Table 2: Accuracy (%) on the Digits dataset.

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</tbody>
</table>
Conclusion
1. New bound of the target risk which unifies weights and representations in UDA.

\[ \epsilon_T(g\phi) \leq \epsilon_{w.s}(g\phi) + 6 \cdot \text{INV}(w, \phi) + 2 \cdot \text{TSF}(w, \phi) + \epsilon_T(f_T\phi) \]
Conclusion

1. New bound of the target risk which unifies weights and representations in UDA.

2. Theoretical analysis of the role of inductive bias when designing both weights and the classifier.

\[
\varepsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{1 - \beta} \left( \varepsilon_S(g_s\varphi) + 6 \cdot \text{INV}(\varphi) + 2 \cdot \hat{\text{TSF}}(\varphi, \tilde{g}) + \varepsilon_T(f_T\varphi) \right)
\]
1. New bound of the target risk which unifies weights and representations in UDA.

2. Theoretical analysis of the role of inductive bias when designing both weights and the classifier.

3. New learning procedure ▷ weak inductive bias can make adaptation more robust even when stressed by strong label shift between source and target domains.

\[
\begin{align*}
\varphi^* &= \arg \min_{\varphi \in \Phi} \varepsilon_{w(\varphi) \cdot S}(g_w \cdot S \varphi) + \lambda \cdot \widehat{\text{TSF}}(w, \varphi, g_w \cdot S) \\
\text{such that } w(\varphi) &= \arg \min_w \text{INV}(w, \varphi)
\end{align*}
\]
1. New bound of the target risk which unifies weights and representations in UDA.

2. Theoretical analysis of the role of inductive bias when designing both weights and the classifier.

3. New learning procedure ▷ weak inductive bias can make adaptation more robust even when stressed by strong label shift between source and target domains.

This work leaves room for in-depth study of stronger inductive bias by providing both theoretical and empirical foundations.
Thank you!
Motivations

\[
\varepsilon_T(g\varphi) \leq \underbrace{\varepsilon_S(g\varphi) + d_G(\varphi)}_{\text{Controllable}} + \underbrace{\lambda_G(\varphi)}_{\text{Not controllable}}
\] (14)

**An unexpected trade-off**

Let \( \psi \) be a representation which is a richer feature extractor than \( \varphi \): \( G \circ \varphi \subset G \circ \psi \). Then,

\[
d_G(\varphi) \leq d_G(\psi) \text{ while } \lambda_G(\psi) \leq \lambda_G(\varphi)
\] (15)
Motivations

Invariance is conflicting with label shift [Zhao et al., ICML2019]

\[ \lambda_g(\varphi) \geq \frac{1}{2} (\text{JS}(Y) - \text{JS}(Z))^2 \]  \hspace{1cm} (16)

If \( \text{JS}(Z) \rightarrow 0 \), \( \lambda_g(\varphi) \) can not be small if \( \text{JS}(Y) \) is high... \( \triangleright \) We should weight the source distributions! But how...

- how to design weights?
- how weights interact with invariance?
- why predictions are important in UDA?
Our strategy

- emerges from the sup / inf duality computed on a small hypothesis class $\mathcal{G} \circ \varphi$.

**Our strategy**
Express both invariance and transferability of representations as supremum over a large space of critic functions

- See the paper for details about property of the critic functions

### 2 critic functions space

- $\mathcal{F} \triangleright \text{'large' function space from } \mathcal{Z} \text{ to } [-1, 1]$
- $\mathcal{F}_C \triangleright \text{'large' function space from } \mathcal{Z} \text{ to } [-1, 1]^C$

- Typically continuous functions.
2 critic functions space

- $\mathcal{F} \triangleright 'large' \text{ function space from } \mathcal{Z} \text{ to } [-1, 1]$
- $\mathcal{F}_C \triangleright 'large' \text{ function space from } \mathcal{Z} \text{ to } [-1, 1]^C$

Typically continuous functions.

2 errors

- captures the difference between source and target distribution of representations:

\[
\text{INV}(\phi) := \sup_{f \in \mathcal{F}} \mathbb{E}_S[f(Z)] - \mathbb{E}_T[f(Z)] \tag{17}
\]

- catches if the coupling between $Z$ and $Y$ shifts across domains:

\[
\text{TSF}(\phi) := \sup_{f \in \mathcal{F}_c} \mathbb{E}_S[Y \cdot f(Z)] - \mathbb{E}_T[Y \cdot f(Z)] \tag{18}
\]
Role of weights

Why designing weights?

- Prediction weighting [Partial Adversarial Domain Adaptation, Cao et al. 2018] ▷ *Estimated labels are used to re-weight the source domain:*

  \[
  w(x) = \frac{p_T(g(z))}{p_S(g(z))}
  \]

- Entropy conditioning [Conditional Adversarial Domain Adaptation, Long et al. 2018] ▷ *Transfer only confident samples:*

  \[
  w(x) \propto 1 + e^{-H(g(z))} \quad \text{where } H \text{ is the entropy.}
  \]

Inductive design of weights

It exists a function \( \psi : Z \rightarrow Z' \), \( \psi(z) =: z' \) s.t. \( w \) is a function of \( Z' \).

Weights enforces new invariance

If the bound is tight, then,

\[
\begin{align*}
  w(z') &= \frac{p_T(z')}{p_S(z')} \quad \text{and} \quad p_S(z|z') = p_T(z|z') \\
\end{align*}
\] (19)
Role of weights

\[ z := \varphi(x) \]

Non-overlapping distributions

\[ z_0 := \psi(z) \]

Deep net

\[ p_T(x) \]

\[ p_T(z) \]

\[ p_S(x) \]

\[ p_S(z) \]

\[ p_S(z|z') = p_T(z|z') \]

\[ z' = g(z) \]

Inductive bias
Detailed view of RUDA

\[ \varepsilon_T(\tilde{g}\varphi) \leq \frac{\beta}{\beta} \{ \varepsilon_w \cdot s(g_w \cdot s\varphi) \]

+ 6 \cdot \text{INV}(w, \varphi) \triangleright \text{Set weight s.t. INV}(w, \varphi) = 0

+ 2 \cdot \text{TSF}(w, \varphi, \tilde{g}) \triangleright \text{Set inductive classifier s.t. } \tilde{g} = g

+ \varepsilon_T(f_T \varphi) \}

- Set weight s.t. INV(w, \varphi) = 0:

\[ w(z) := \frac{p_T(z)}{p_S(z)} = \frac{1 - d(z)}{d(z)} \quad (20) \]

where \( d \) is a domain classifier (trained to map 1 in the source domain and 0 in the target domain.)

- Set inductive classifier s.t. \( \tilde{g} = g: \beta = 1 \)

"This is a weak inductive design (\( \beta = 1 \)), thus, theoretical guarantee from bound 4 is not applicable. However, there is empirical evidence that showed that predicted labels help in UDA"
\[
\begin{align*}
\theta^*_\varphi & = \arg\min_{\theta_\varphi} \mathcal{L}_c(\theta_g, \theta_\varphi | \theta_d) + \lambda \cdot \mathcal{L}_{\text{TSF}}(\theta_\varphi, \theta_d | \theta_d, \theta_g) \\
\theta_g & = \arg\min_{\theta_g} \mathcal{L}_c(\theta_g, \theta_\varphi | \theta_d) \\
\theta_d & = \arg\min_{\theta_d} \mathcal{L}_{\text{INV}}(\theta_d | \theta_\varphi)
\end{align*}
\] (21)

\[
\begin{align*}
\varphi^* & = \arg\min_{\varphi \in \Phi} \varepsilon_{w(\varphi)} \cdot S(g_w \cdot S\varphi) + \lambda \cdot \widehat{\text{TSF}}(w, \varphi, g_w \cdot S) \\
such that \ w(\varphi) & = \arg\min_{w} \text{INV}(w, \varphi)
\end{align*}
\] (RUDA)

Detailed view of RUDA
$L_{TSF} := \mathbb{E}_S[w(z)Y \cdot \log(d(z))] + \mathbb{E}_T[g(z) \cdot \log(1 - d(z))]$

\[ w(z) := \frac{1 - d(z)}{d(z)} \]

\[ g(z) \]

$d(z)$

$L_{INV} := \mathbb{E}_S[\log(d(z))] + \mathbb{E}_T[\log(1 - d(z))]$